15.9 Videos Guide

15.9a

Exercise:

- Find the image of the set S under the given transformation.
  S is the square bounded by the lines $u = 0, u = 1, v = 0, v = 1$;
  $x = v, y = u(1 + v^2)$

15.9b

- The Jacobian
  - For a transformation T that maps S in the uv-plane onto R in the xy-plane with inverse transformation T^{-1} (which maps R onto S), the Jacobian of x and y with respect to u and v is
    $$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix}$$
- Change of variables for double integrals (under T)
  - $\iint_R f(x,y)\,dA = \iint_S f\big(g(u,v),h(u,v)\big)\left|\frac{\partial(x,y)}{\partial(u,v)}\right|\,dudv$

Exercises:

15.9c

- Use the given transformation to evaluate the integral.
  $\iint_R (4x + 8y)\,dA$, where R is the parallelogram with vertices $(-1,3)$, $(1,-3)$, $(3,-1)$, and $(1,5)$;
  $x = \frac{1}{4}(u + v), y = \frac{1}{4}(u - 3v)$

15.9d

- A region R in the xy-plane is given. Find equations for a transformation T that maps a rectangular region S in the uv-plane onto R, where the sides of S are parallel to the u- and v-axes.
  R is bounded by the hyperbolas $y = 1/x$, $y = 4/x$ and the lines $y = x$, $y = 4x$ in the first quadrant

15.9e

- Evaluate the integral by making an appropriate change of variables.
  $\iint_R (x = y)e^{x^2-y^2}\,dA$, where R is the rectangle enclosed by the lines $x - y = 0$, $x - y = 2$, $x + y = 0$, and $x + y = 3$

- Using $T: S \to R$ and $T^{-1}: R \to S$

- $u = f(x, y)$ and $v = g(x, y)$ are useful in finding limits of integration
- $x = x(u, v)$ and $y = y(u, v)$ are useful for finding $\frac{\partial(x,y)}{\partial(u,v)}$
- Substitute using whichever is most convenient